Evelyne Hubert (Inria Côte d'Azur) : *Explicit semi-algebraic description of the orbit space of Weyl group actions*

The action of the Weyl group on the compact or algebraic torus arise in the representation theory of compact Lie groups or reductive groups. Its invariants are connected with the characters and form a polynomial algebra. This action is also described on lattices associated to a crytallographic root system. As such Weyl groups arise in a number of other domains like Fourier analysis, codes or combinatorics.

For the infinite families of root systems, those of types An-1, Bn, Cn and Dn (also G2 actually), the Weyl group is a semi direct product of a group {1,-1}^k with the symmetric group Sn. Exploiting this decomposition we make explicit a polynomial matrix whose locus of positivity is the image of the compact torus by the fundamental invariants, the orbit space. Remarkably it is a unified formula for all the above types when written in terms of the generalized Chebyshev polynomials associated to the root system.

This explicit formula is an essential ingredient in our approach to optimize trigonometric polynomials with crytallographic symmetry. The problem can be reduced to polynomial optimization on a semi-algebraic set, a subject that has ripened in the last two decades after the seminal article of Lasserre. We applied our approach to compute the spectral bound for the chromatic number of some infinite set-avoiding graphs (a.k.a. Caley graphs).

This is joint work with Tobias Metzlaff, Philippe Moustrou and Cordian Riener.

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